High-Performance Logarithmic Converters Design

高效能對數轉換器設計

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Outline

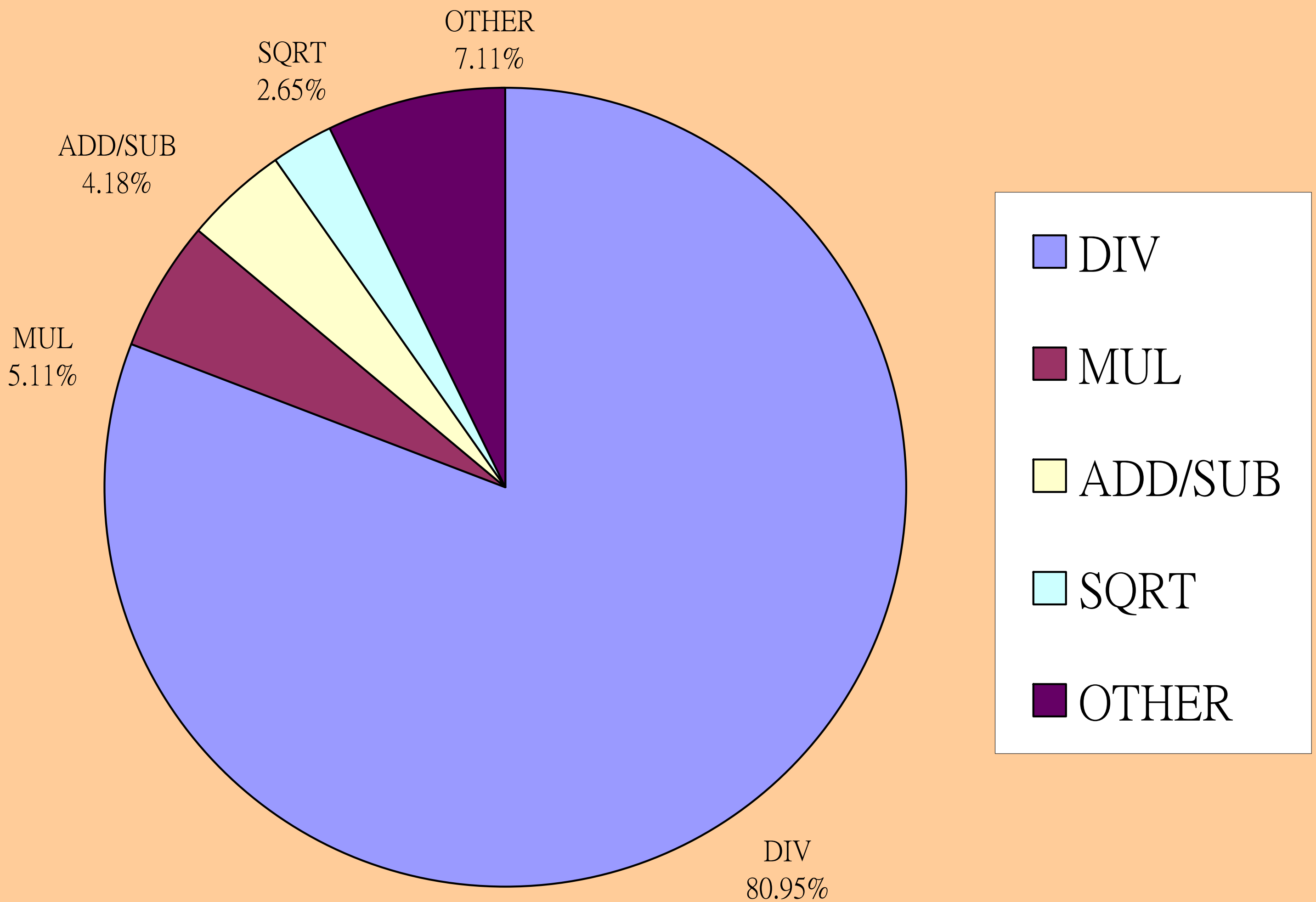
- Introduction
- Previous work of Logarithmic Converters
  - Mitchell's logarithmic approximation
  - Combet and Sangregory's logarithmic method
  - Abed and Kim's logarithmic approximation
- Proposed Methods
- Simulation Results and VLSI Implementation
- Conclusion

Introduction

Application

- ✓ Digital Processing Processing
- ✓ Three Dimension (3D) Computer Graphic
- ✓ Mobile Cloud Computing
- ✓ Telecommunication

Motivation



OTHER
7.11%
SQRT
2.65%
ADD/SUB
4.18%
MUL
5.11%
DIV
80.95%
DIV
MUL
ADD/SUB
SQRT
OTHER

Motivation

(表 2.1) 對數系統運算轉換表[14]

Operation		Normal Arithmetic	Logarithmic Arithmetic
Multiplication	MUL	$z = x \cdot y$	$X + Y$
Division	DIV	$z = x / y$	$X - Y$
Reciprocal	RCP	$z = 1/x$	$-X$
Square Root	SQRT	$z = \sqrt{x}$	$X \gg 1$
Reciprocal	RSQ	$z = 1/\sqrt{x}$	$-X \gg 1$
Square	SQR	$z = x^2$	$X \ll 1$
Powering	POW	$z = x^y$	$Y \bullet \log_2 X$
Addition	ADD	$z = x + y$	$X + \log_2(1 + 2^{Y-X})$
Subtraction	SUB	$z = x - y$	$X + \log_2(1 - 2^{Y-X})$

Examples

Taking Examples:

(1) $32 \times 16 = 512$

$Log_2(32) = 5$
$Log_2(16) = 4$
$5 + 4 = 9$
$2^9 = 512$

(2) $32 / 16 = 2$

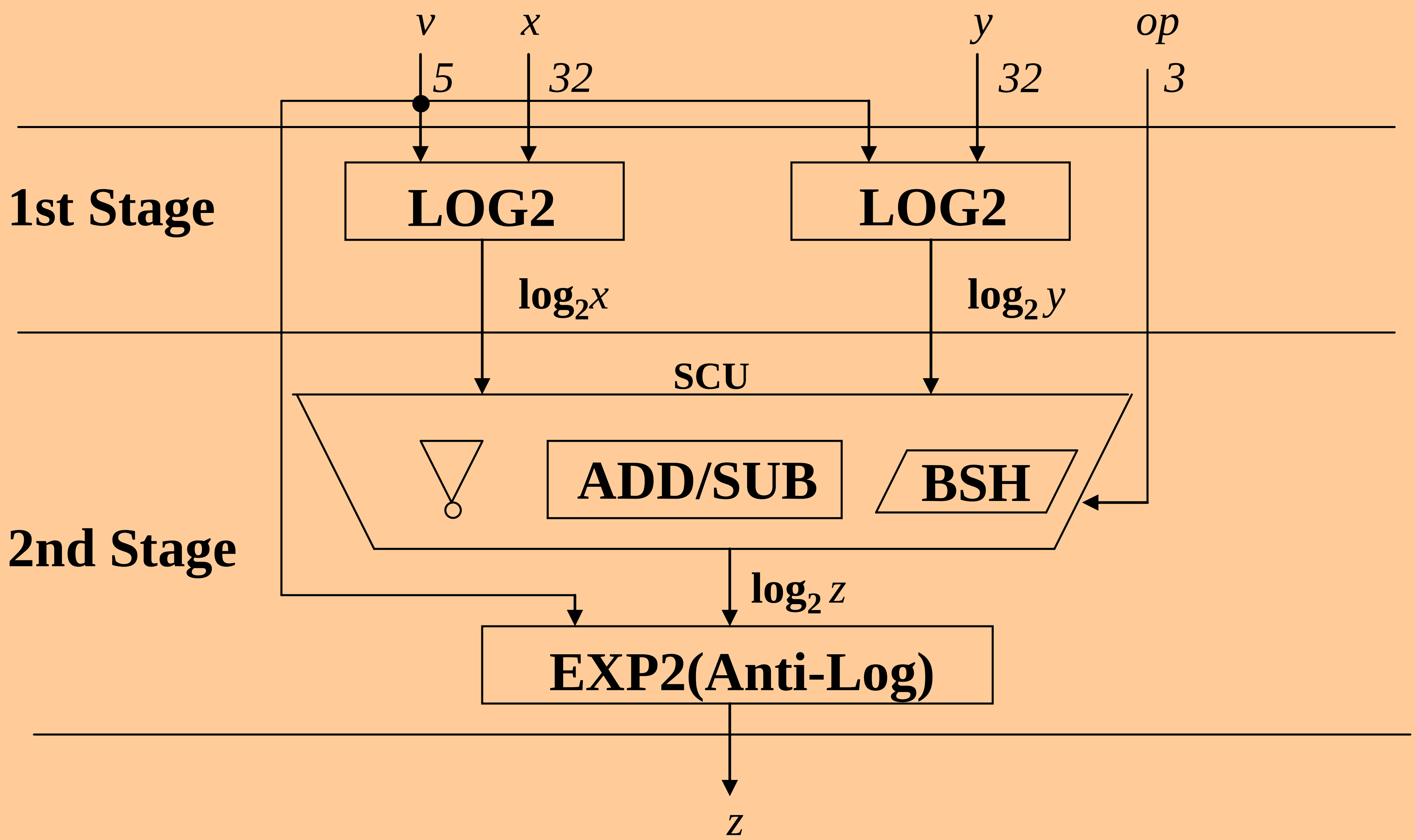
$Log_2(32) = 5$
$Log_2(16) = 4$
$5 - 4 = 1$
$2^1 = 2$

(3) $16^2 = 256$

$Log_2(16) = 4$, $4 << 1 = 4 * 2 = 8$

$2^8 = 256$

Architecture of Logarithm- Based Unit

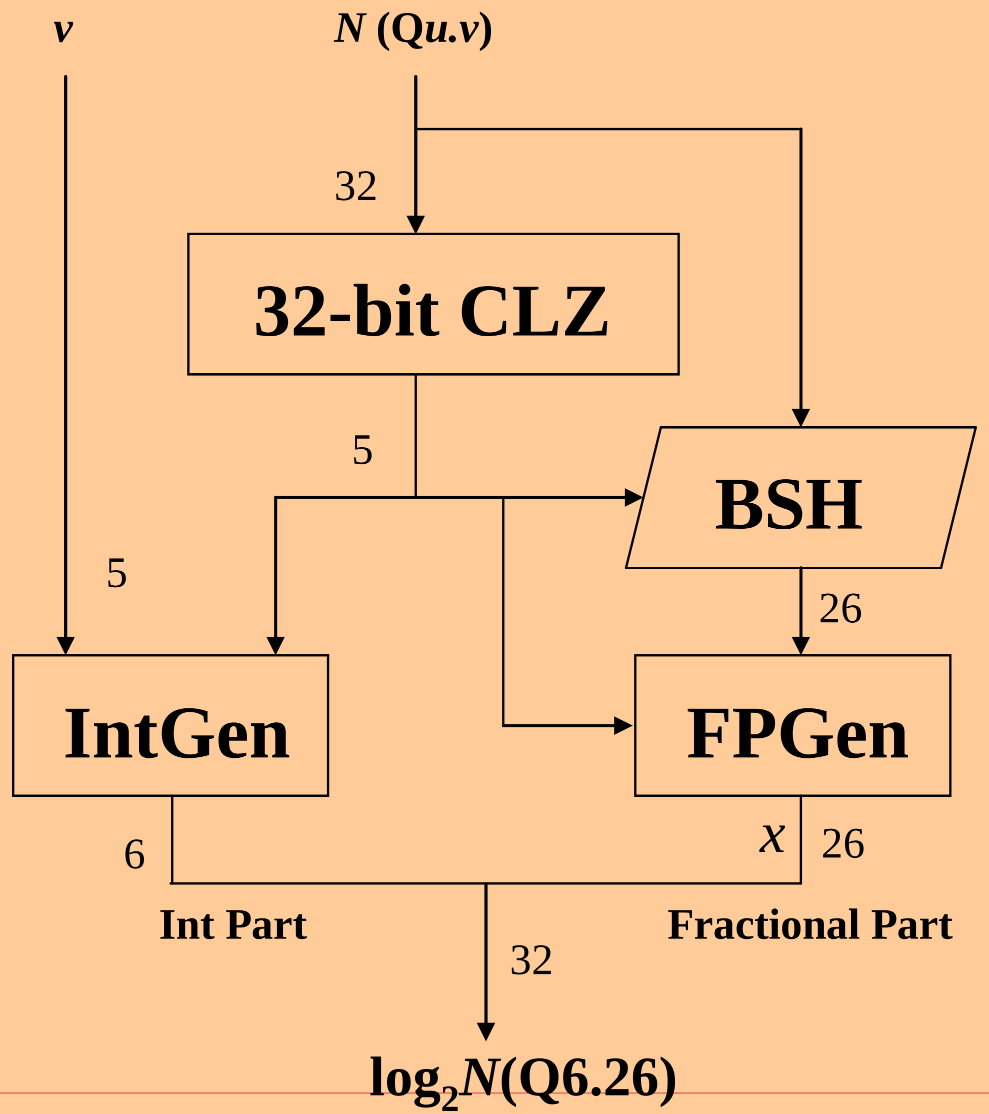


Components of Logarithm- Based Unit

Three components:

- Two Logarithmic Conversion Units
- One Calculation Unit
- One Antilogarithmic Unit

Architecture of 1st Stage Logarithm-Based Unit



The input/output format of 2-based logarithmic units

Bit Index	x_u	…	x_1	x_0	x_{-1}	x_{-2}	…	$x_{-(v-1)}$	x_{-v}
Value	2^u	…	2^1	2^0	2^{-1}	2^{-2}	…	$2^{-(v-1)}$	2^{-v}

Fraction point

$Q\,(u\,,\,v)$	u-bit Integer part	v-bit Fractional part

Fixed-point number representations for a number Q(u,v)

Bit Index	x_5	…	x_1	x_0	x_{-1}	x_{-2}	…	x_{-25}	x_{-26}
Value	2^5	…	2^1	2^0	2^{-1}	2^{-2}	…	2^{-25}	2^{-26}

Fraction point

$Q\,(6\,,\,26)$	*6*-bit Integer part	*26*-bit Fractional part

Example for the number Q(6,26)

Previous Methods of Logarithmic Converters

Previous Methods

- ✓ **Look-Up Table**

  the area will increase exponentially as the bit-width of input, but the delay is shorter

- ✓ **Iterative**

  smaller area, high accuracy with longer delay

- ✓ **Shift-and- Add**

  tradeoffs the area, delay and accuracy, using simple equation to approximate the real Log.

Shift-and-Add Methods

Mitchell's one-region Method in 1962

$N = 2^k (1+ x)$

ex. $N = 6 = 110_2 = 2^2 (1+0.10_2)$

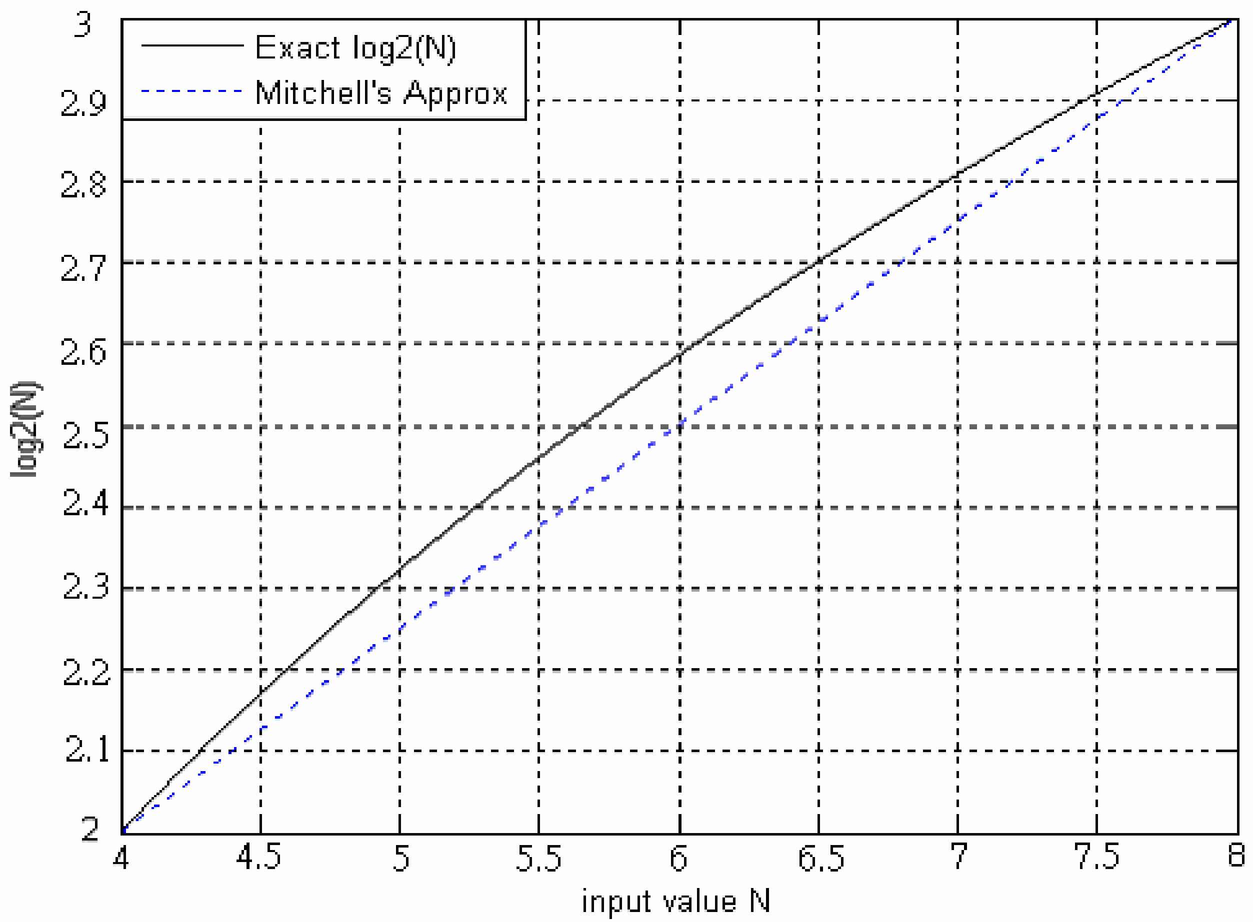
$\text{Log}_2(N) = k + \text{Log}_2(1+x)$ (Exact Log_2(N))

$\text{Log}_2(N)_{\text{Mitchell}} = k + x$ (Mitchell)

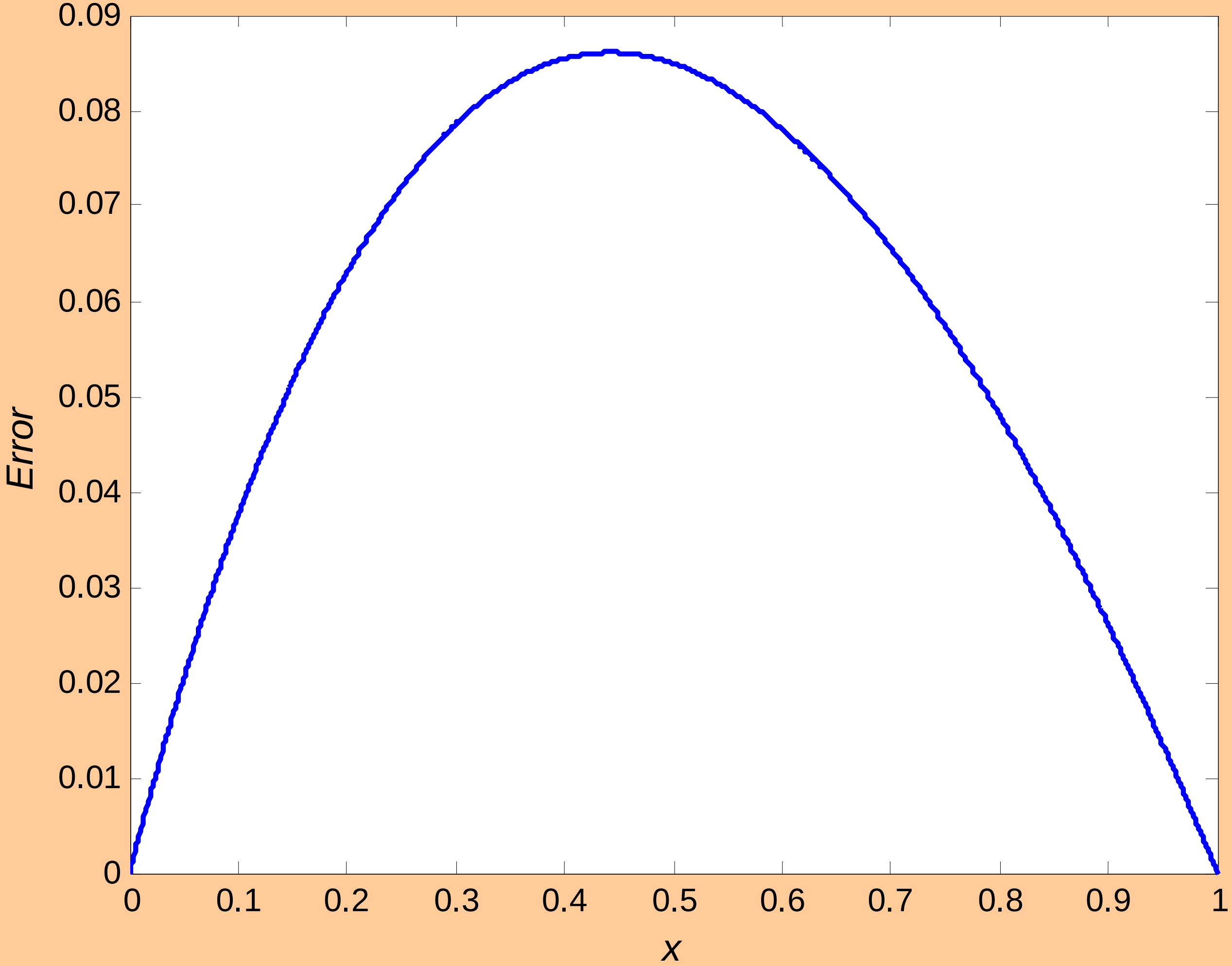
ex. $\text{Log}_2(6)_{\text{Mitchell}} = 2+0.10_2 = 10.10_2$

$$\text{Error}(N) = \text{Log}_2(N) - \text{Log}_2(N)_{\text{Mitchell}}$$
$$= \text{Log}_2(1+x) - x$$

The Approximations of Mitchell's one-region Method



Error Analysis of Mitchell's one-region Method



The Approximations of Combet's four-region Method in 1965

$$\log_2(N) = k + \log_2(1 + x)$$

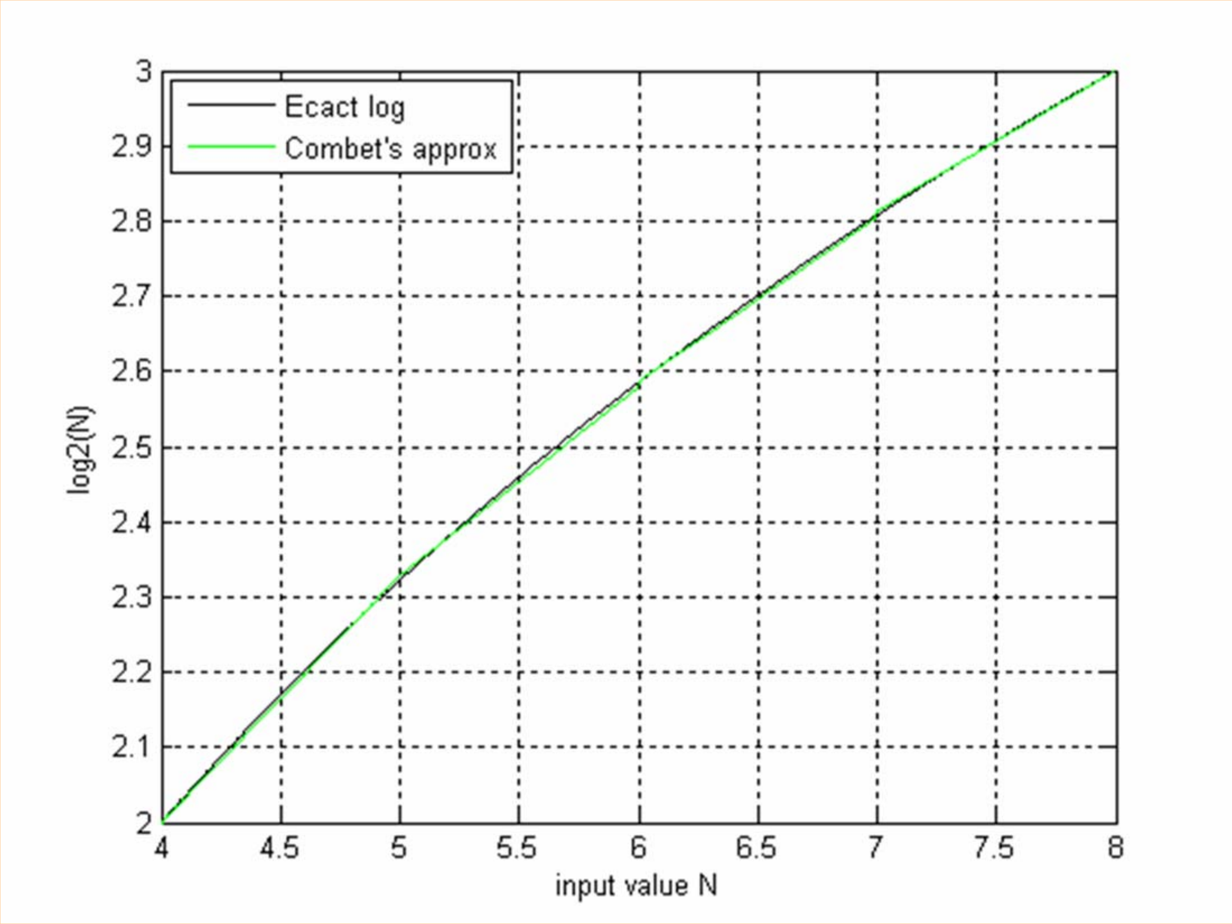
$$\log_{2Combet}(1+x) = x + \frac{5}{16}x$$ for x [0,0.25)

$$\log_{2Combet}(1+x) = x + \frac{5}{64}$$ for x [0.25,0.5)

$$\log_{2Combet}(1+x) = x + \frac{1}{8}\overline{x} + \frac{3}{128}$$ for x [0.5,0.75)

$$\log_{2Combet}(1+x) = x + \frac{1}{4}\overline{x}$$ for x [0.75,1)

The Approximations of Combet’s four-region Method



The Approximations of Sangregory's two-region Method in 1999

$$\log_2(N) = k + \log_2(1 + x)$$

$$\log_{2Sangregory}(1+x) = x + \frac{1}{4} \cdot x_{4MSBits}$$ **for *x* [0,0.5)**

$$\log_{2Sangregory}(1+x) = x + \frac{1}{4} \cdot \overline{x}_{4MSBits}$$ **for *x* [0.5, 1)**

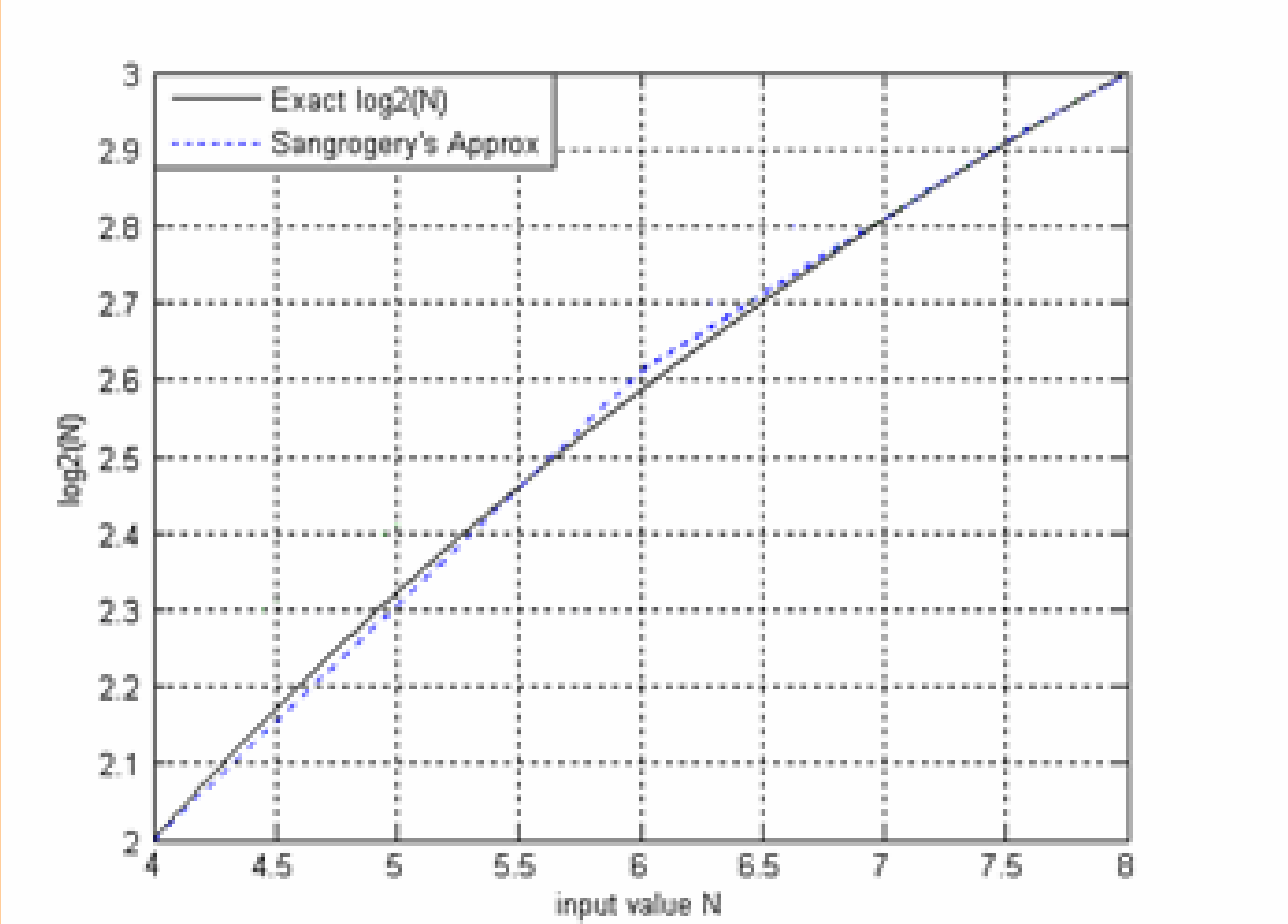
The Approximations of Abed's two-region Method in 2003

$$\log_2(N) = k + \log_2(1 + x)$$

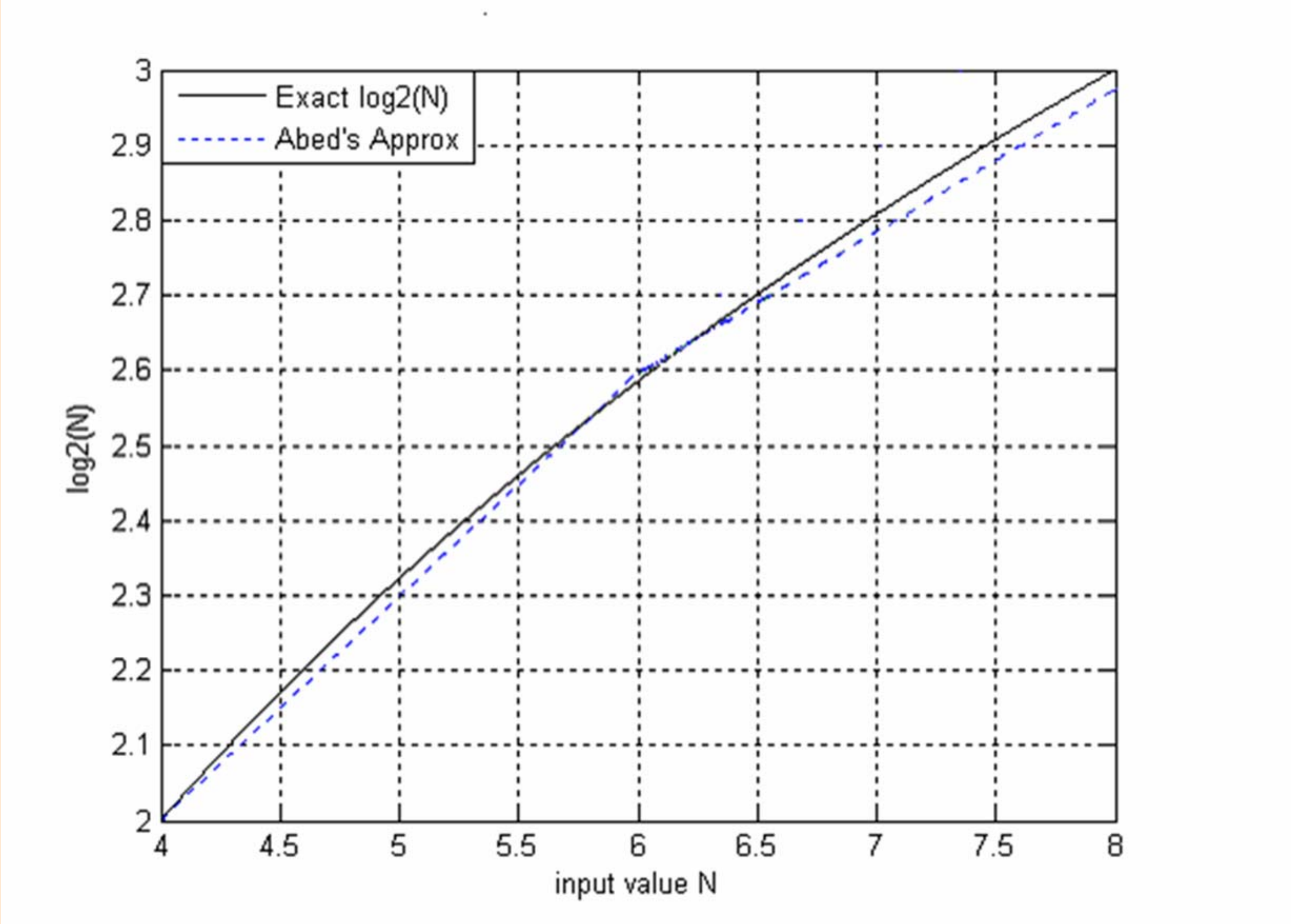
$$\log_{2Abed}(1 + x) = x + \frac{1}{4} \cdot x_{3MSBits}$$ **for *x* [0,0.5)**

$$\log_{2Abed}(1 + x) = x + \frac{1}{4} \cdot \bar{x}_{3MSBits}$$ **for *x* [0.5, 1)**

The Approximations of Sangregory's two-region Method



The Approximations of Abed’s two-region Method



The Approximations of Kim's eight-region Method in 2008

$$\log_{2Hoi-Jun}(1+x) = x + \frac{1}{2}x + \frac{1}{8}\overline{x} + \frac{1}{128}\overline{x}$$ for x [0, 0.125)

$$\log_{2Hoi-Jun}(1+x) = x + \frac{1}{2}x + \frac{1}{4}\overline{x} + \frac{1}{64}x + \frac{15}{1024}$$ for x [0.125, 0.25)

$$\log_{2Hoi-Jun}(1+x) = x + \frac{1}{2}x + \frac{1}{8}\overline{x} + \frac{1}{64}\overline{x} + \frac{46}{1024}$$ for x [0.25, 0.375)

$$\log_{2Hoi-Jun}(1+x) = x + \frac{1}{128}\overline{x} + \frac{91}{1024}$$ for x [0.375, 0.5)

$$\log_{2Hoi-Jun}(1+x) = x + \frac{1}{8}\overline{x} + \frac{1}{16}x + \frac{1}{128}\overline{x} + \frac{123}{1024}$$ for x [0.5, 0.625)

$$\log_{2Hoi-Jun}(1+x) = x + \frac{1}{8}\overline{x} + \frac{1}{64}\overline{x} + \frac{167}{1024}$$ for x [0.625, 0.75)

$$\log_{2Hoi-Jun}(1+x) = x + \frac{1}{4}\overline{x} + \frac{1}{16}x + \frac{1}{64}\overline{x} + \frac{215}{1024}$$ for x [0.75, 0.875)

$$\log_{2Hoi-Jun}(1+x) = x + \frac{1}{4}\overline{x} + \frac{1}{128}\overline{x} + \frac{264}{1024}$$ for x [0.875, 1.0)

Proposed Logarithmic Conversion Using Binary Searching Schemes

Proposed Algorithm

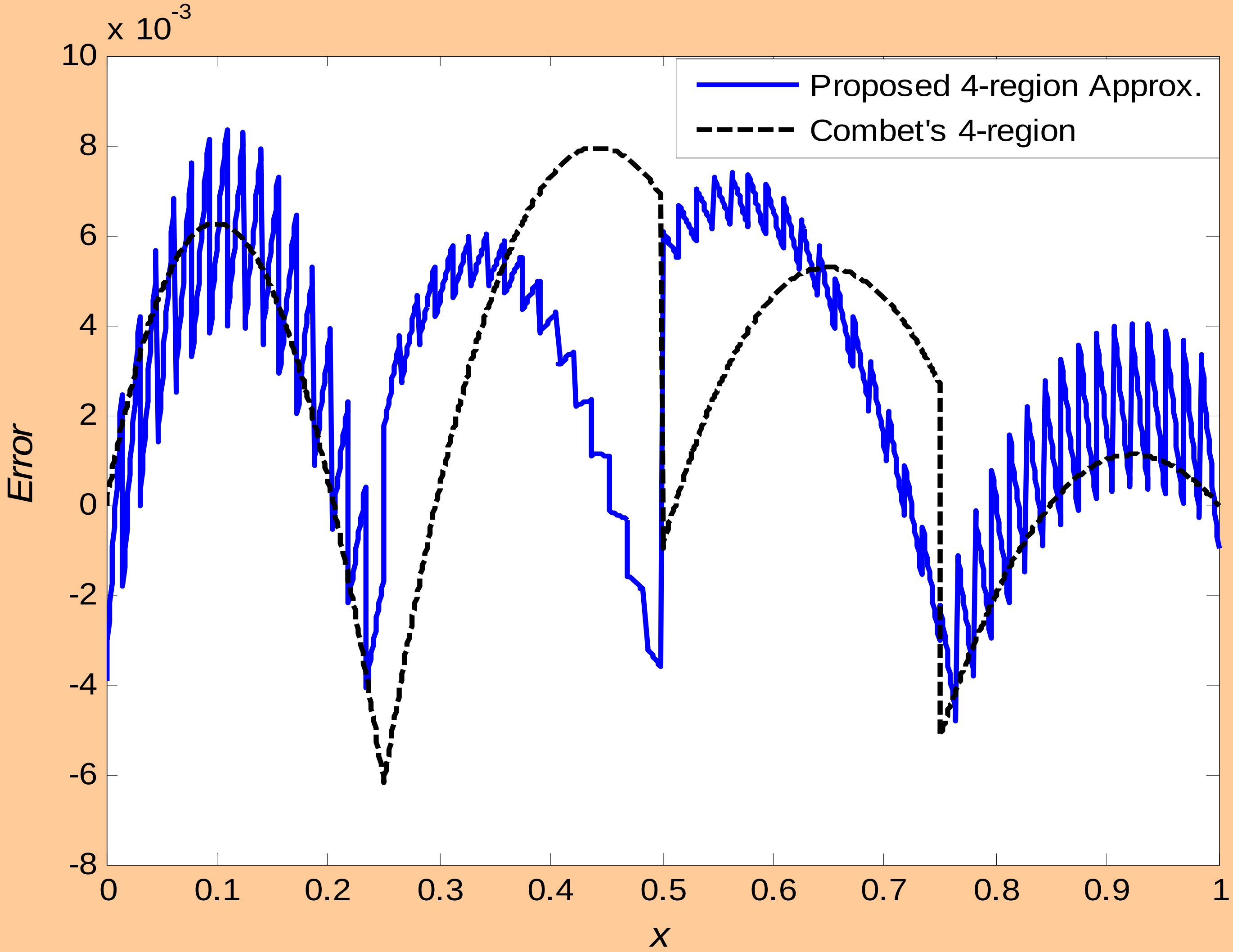
$$\log_2(1+x) \approx Y_{proposed} = x + ax + b$$

$$Error_{\Pr oposed} = \log_2(1+x) - (x + ax + b)$$

[Input] N regions of x ($0 \le x < 1$)
For each region i ($1 \le i \le N$) of x do
For a_i = -2 to 2 step 1/512 do
For b_i = 0 to 1 step 1/2048 do
Find the values of (a_i, b_i) which can produce the minimum total error range among approximation errors for each region i.
End for
End for
End for
[Output] N value sets (a_i, b_i) of x.

Fig. 6 Proposed binary coefficient searching algorithm for obtaining the values set (a, b) in each region.

Error analysis of our proposed 4-region logarithmic converter compared with Combet 4-region method



Area delay and error estimations of our proposed 4-, 8-, and 16-region logarithmic conversions

Items	Proposed Work		
No. of Region	4	8	16
Maximum Positive Error	0.0083	0.00092	0.00013
Maximum Negative Error	-0.0048	-0.002	-0.00103
Total Error Range	0.0131	0.00292	0.00116
Area(um^2)	9985.85	27975.02	37548.40
Delay(ns)	2.1ns	3.5ns	4.01ns

Conclusions

- Proposed Low-error and ROM-Free Logarithmic Converters Using Shift-and-Add Operation.
- Our Proposed Converter can apply to real-time DSP and 3D Graphics.

Thank you for your attention

Q & A

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